

A new statistic to detect segmentation or unequal variance in 2-Alternative Choice (2-AC) testing

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DTU Informatics

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Paired preference testing

2 products :

A Chocolate bar (standard)

B Chocolate bar with darker chocolate

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2-Alternative Forced Choice (2-AFC):

- Do you prefer A or B?

Prefer A Prefer B

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2-Alternative Forced Choice (2-AFC):

- Do you prefer A or B?

Prefer A Prefer B

2-Alternative ~~Forced~~ Choice (2-AC):

- Do you prefer A or B, **or do you have no preference?**

Prefer A No Preference Prefer B

Paired preference with a *no preference* option

Terminology:

Paired preference with a *no preference* option

Terminology: *No preference* \sim *No difference* \sim Ties

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Why allow for a *no preference* option?

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Why avoid a *no preference* option?

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Why avoid a *no preference* option?

- Statistical methods less well-known

Placebo experiments and identity norms

Consider the data:

	Prefer A	No Preference	Prefer B	Total
All counts	90	20	90	200

Placebo experiments and identity norms

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Segment 1	8	10	82	100
Segment 2	82	10	8	100

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Ennis and Ennis (2012) suggest:

- 1 Perform placebo experiment
- 2 Estimate the *identity norm*:

The expected proportion of counts for identical products

Ennis, D. M. and J. M. Ennis (2012). Accounting for no difference/preference responses or ties in choice experiments. *Food Quality and Preference* 23, 13-17.

Example: Comparing data with an identity norm

Ennis' Approach:

	Prefer A	No Preference	Prefer B	Total
Data	25	15	60	100
Identity norm	0.4	0.2	0.4	—

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$$\begin{aligned}X_2^2 &= (25 - 40)^2/40 + (15 - 20)^2/20 + (60 - 40)^2/40 \\ &= 5.625 + 1.250 + 10.00 = 16.875\end{aligned}$$

$$p\text{-value} = 0.00022$$

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- Assumes identity norm known without error
- Uncertainty in the placebo experiment not taken into account!

Example: Comparing data with an identity norm

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Assume $n = 100$ in placebo experiment:

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Expected counts:

	Prefer A	No Preference	Prefer B	Total
Data	32.5	17.5	50	100
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The standard (genuine) Pearson χ^2 test:

$$X_2^2 = (25 - 32.5)^2/32.5 + (40 - 32.5)^2/32.5 + \dots + (40 - 50)^2/50 = 8.18$$

p -value = 0.0168 (previous p -value = 0.00022)

Effect of sample size in placebo experiment

Standard Pearson test on 2×3 table:

n_{placebo}	χ_2^2 statistic	p -value
20	2.80	0.24619
50	5.50	0.06393
100	8.18	0.01677
1000	15.15	0.00051
10^9	16.87	0.00022

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$\chi^2 = 16.87$ and p -value = 0.00022

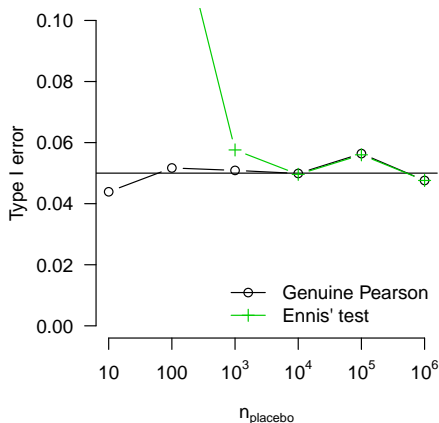
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- Insightful interpretation
- Easy to compute

Approach

- 1 Consider 5 test statistics
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Tie effects	$s_0 = s_1$	$s_0 \neq s_1$	1

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Pooled Test	$s_0 = s_1$ and $p_1 = 0.5$	$s_0 \neq s_1$ or $p_1 \neq 0.5$	2

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Settings for power simulations

Placebo experiment (true identity norm):

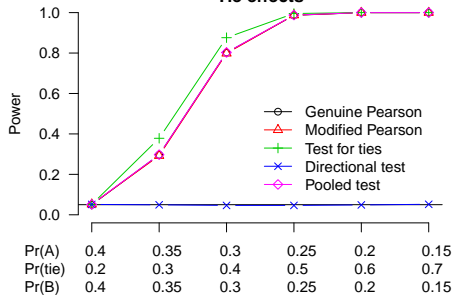
Prefer A	No Preference	Prefer B
0.4	0.2	0.4

Power simulations in 6 settings:

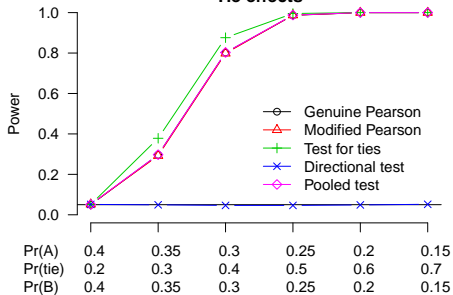
Placebo sample size	Structures in preference data		
	Tie effects	Directional effects	Joint effects
100	1A	1B	1C
1.000.000	2A	2B	2C

- $n_{preference} = 100$
- 10.000 simulations at each point

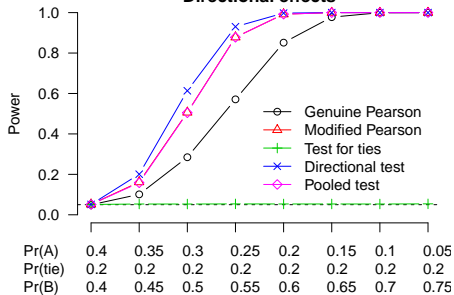
Tie effects



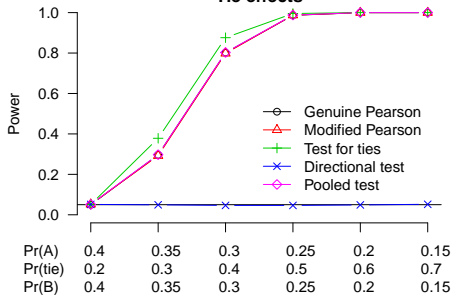
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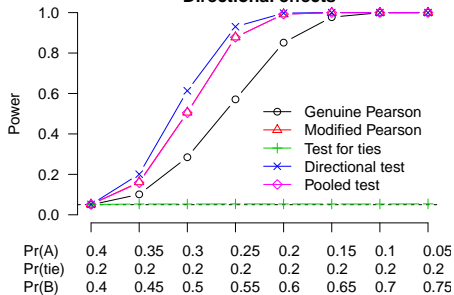
Directional effects



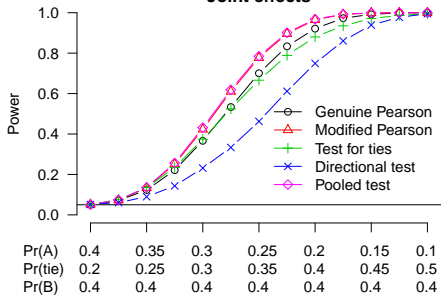
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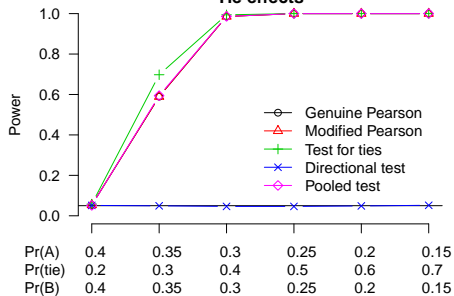
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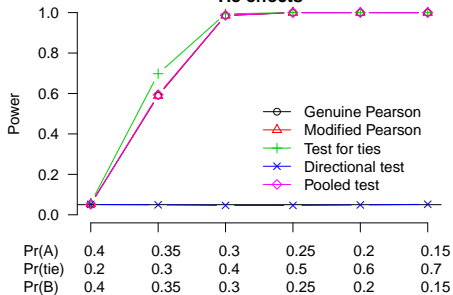
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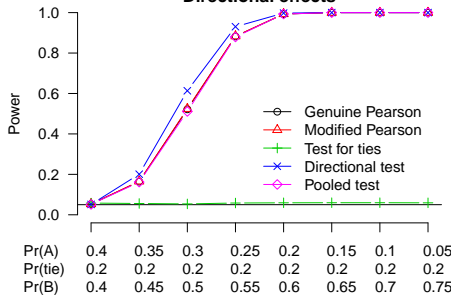
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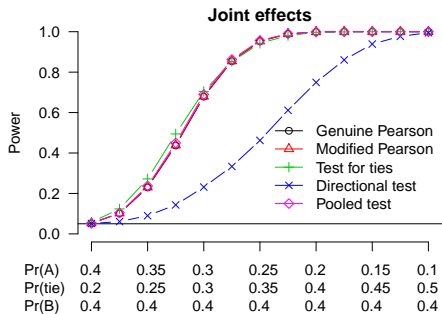
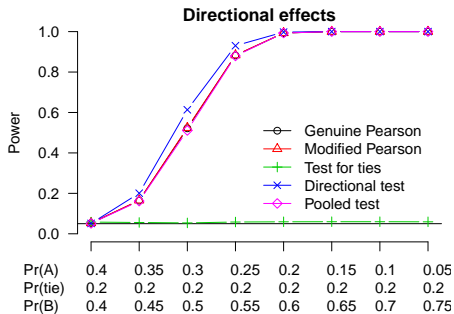
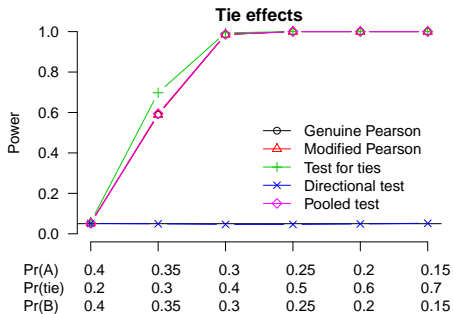


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ANOVA-like analysis:

Test	χ^2	df	<i>p</i> -value
Pooled test	7.00	2	0.030
Tie effects	4.78	1	0.029
Directional effects	2.23	1	0.136

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Modified Pearson	7.20	2	0.027

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 - Segmentation
 - Heterogeneity in preference
 - Unequal variances in the underlying perceptual distributions

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